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Valuing executive stock options with endogenous departure

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Abstract

Executive stock options differ from exchange-traded options because of vesting and portability restrictions. Executive departure from the firm forces early exercise, reducing the value of executive options. Current methodology calculates the option value by multiplying the Black–Scholes option price by the departure probability. This ignores the possibility that executive departure is less likely when stock price is high, and thus is correlated with the stock price. We show that this correlation implies a substantial increase in option values. A similar situation occurs in performance-based option packages, where the actual number of options granted depends on stock performance.

Key words: Management compensation; Executive stock options; Valuation

JEL classification: G12; G13; M41

1. Introduction

As executive stock options become more frequently used in compensation packages, the Financial Accounting Standards Board (FASB) has considered rules that would impose a new accounting charge on executive stock options (ESOs). This proposal sparked a heated debate as to whether corporations should expense stock options and as to the appropriate option valuation model.

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Financial economists have long used standard valuation models to price options, and recognize that options, whether traded or not, are valuable assets. The FASB position is also that 'nonrecognition of compensation cost implies either that ESOs are free to employees or that options have no value – neither of which is true'. Because options have value both to employers and employees, it seems justified to require companies to recognize an expense for all stock-based compensation awards. Furthermore, the advent of inexpensive computing power considerably facilitates the computation of fair option values.

Since executive options bestow the right to buy the stock at a pre-arranged price, they essentially are call options. One method for valuing ESOs under the FASB proposal is based on the Black–Scholes (BS) option pricing model (Black and Scholes, 1973) that has been successfully applied to a variety of financial markets. The BS option pricing model can be applied to find the price of a European call option on a non-dividend-paying, traded stock whose price follows a geometric Brownian motion process. For American options, or options on assets paying discrete dividends, the option price can be found using the binomial model developed by Cox, Ross, and Rubinstein (1979), a numerical approach that includes the BS model as one of its limits.

The success of the BS option pricing model motivates its application to the valuation of executive options. However, ESOs differ from standard stock options because of vesting and portability restrictions, and may additionally be performance-based. These differences have been recognized in the literature,[1] and simple models have been proposed to account for the stochastic nature of ESO lives. Because some employees are likely to leave the firm before vesting, a proposed adjustment method, put forth in FASB exposure draft 127-C (1993), multiplies the usual value of granted options by the number of employees that are *expected* to stay.

This approach, however, ignores a key feature of executive options: an employee's decision to leave and an employer's decision to terminate are both typically correlated with the stock price.[2] ESOs give executives an incentive to stay precisely in those states of the world where the stock price, and thus the option price, is high. Similarly, poor executive performance, reflected in the stock price, is more likely to lead to firing. As this paper demonstrates, this positive correlation between the number of options exercised and the stock price yields option values substantially higher than previously recognized. Lambert et al. (1991) and Huddart (1994) emphasize a different feature of ESOs, the inability of the executive to trade either the option or the underlying stock freely. A risk-averse executive may therefore choose to exercise ESOs early, lowering the option value. A complete analysis of ESOs should consider both of these effects.

[1] See, for instance, Noreen and Wolfson (1981) and Foster et al. (1991).

[2] This was also noted by Jennergren and Näslund (1993).

This paper is organized as follows. Executive stock options are described in Section 2. The impact of vesting restrictions is examined in Section 3. Section 4 examines the pricing of ESOs when the granting of options is based on firm performance. The closed-form solutions of these sections are complemented in Section 5 by a numerical approach that recognizes the early exercise feature more fully. Section 6 concludes.

2. Executive stock options

The BS option pricing model applies to traded European options; option price is a function of the underlying stock price S_t, option strike price K, interest rate r, underlying asset volatility σ, and option life τ. Executive stock options may differ from traded stock options for the following reasons:

1. *Nonportability*: If the executive leaves, the option must be immediately exercised or lost. The option value is then $f_t = \max(S_t - K, 0)$, at time $t < \tau$. This feature potentially reduces the value of the option, since exercise effectively wipes out any remaining time value in the option. The option is reduced to its immediate exercise value, which may be much less than the theoretical price of a traded option.

2. *Vesting*: During a vesting period of length τ_1, the option is forfeited if the executive leaves: $f_t = 0$, $t < \tau_1$. This feature may further reduce the value of the option, as an executive departing before vesting loses the value of the option.

3. *Performance-based*: Executive options may be either *fixed* or *performance* stock options. Fixed stock options require only that the executive remain employed by the issuer until the end of the vesting period. Performance-based, or variable, stock options include additional requirements, for example, that the employer earn a given minimum return during the vesting period. Alternatively, the number of options may be a function of earnings or product market share growth.[3] The value of a performance-based package (and the number of options actually vested) may thus depend on the state of nature during the vesting period.

[3] Easton et al. (1992) have shown that, over long intervals, accounting earnings explain most of stock returns. As a result, because of the long-term nature of ESOs, requirements linked to earnings are closely related to stock prices.

Jennergren and Näslund (1993) recently proposed a modification to the BS model to account for executive departure. If executives leave according to an independent Poisson process, at an *exogenous* instantaneous rate λ, they show (assuming the risk of departure is not priced) that the nonportable European ESO value is[4]

$$f' = e^{-\lambda\tau} c(S_0; K, r, \sigma, \tau). \tag{1}$$

Note that this simply multiplies the BS price c by the probability the executive will stay. For example, for a ten-year nonportable European ESO, a 5% annual departure rate implies that f' is approximately 60% of the BS value. Similarly, using a numerical model, they calculate that for a typical ten-year nonportable American ESO with three-year vesting, a 5% annual departure rate implies that f' is 82% of the traded option value. FASB exposure draft 127-C also allows adjustments to the BS price. For vesting, it multiplies the BS price by the probability of staying over the vesting period; for nonportability, it replaces option life τ with expected time-to-exercise; for performance-based options, it multiplies by a second factor, the probability of achieving the required target.

One drawback of such valuation approaches is their assumption of an exogenous departure probability. However, both the probability of an executive choosing to leave a firm and the probability of the firm terminating the executive are related to firm performance, and hence the stock price. Empirically, Coughlan and Schmidt (1985) and Warner et al. (1988) find that executive turnover is negatively related to stock performance.[5] We therefore consider ESO valuation when departure probabilities are negatively correlated with stock price.

3. Vesting restrictions

Suppose a number of European call options on a non-dividend-paying stock (whose price follows geometric Brownian motion) are given (with portability) at vesting. Let τ_1 be the time until vesting, and τ_2 be the time from vesting to expiration. The number of call options *expected* to be granted at vesting is $q(S)$,

[4]We use the notation f' to denote any ESO valuation which does not recognize a correlation between the stock price and the expected number of options exercised.

[5]Coughlan and Schmidt (1985) find an average annual turnover of 8.9%. Warner et al. (1988), restricting the sample to 'forced departures', find a rate of 2.0%.

conditional on S, the stock price at vesting. By risk-neutral valuation,[6] the value of the total package is

$$f = \mathrm{e}^{-r\tau_1}\mathrm{E}^*[q(S)\, c(S;K, r, \sigma, \tau_2)], \tag{2}$$

where E^* denotes the usual risk-neutral expectation.

A special case of (2) is a European ESO where vesting occurs at expiration (equivalently, a nonportable European ESO), and the probability of remaining employed by the firm depends on the stock price at that time. Here, $\tau_1 = \tau$, $c = \max(S - K, 0)$, and $q(S)$ is the probability of staying employed by the firm until expiration, contingent on the level of the stock price, and thus

$$f = \mathrm{e}^{-r\tau}\mathrm{E}^*[q(S)\cdot \max(S - K, 0)]. \tag{3}$$

A closed-form solution can be obtained for a particularly simple form of the probability of staying:

$$q(S) = (S/\bar{S}) \quad \text{for} \quad S \leqslant \bar{S},$$
$$q(S) = 1 \quad \text{for} \quad S > \bar{S}, \tag{4}$$

with $\bar{S} > K$ (for $\bar{S} \leqslant K$, the problem reduces to the BS model).

To value this correctly, integrate (3) using (4):

$$f = \mathrm{e}^{-r\tau}\left[\int_K^{\bar{S}} (S/\bar{S})(S - K)\, g(S)\mathrm{d}S + \int_{\bar{S}}^{\infty} (S - K)g(S)\mathrm{d}S\right], \tag{5}$$

where g is the risk-neutral density of S. After manipulation,

$$f = (K/\bar{S})\, c(S_0; \bar{S}, r, \sigma, \tau)$$
$$+ (S_0/\bar{S})\, e^{(r+\sigma^2)\tau}[c(S_0; K, r + \sigma^2, \sigma, \tau) - c(S_0; \bar{S}, r + \sigma^2, \sigma, \tau)]. \tag{6}$$

Note that the term in brackets is positive, since the call value is decreasing in the strike price, and $\bar{S} > K$. In the limiting case of $\bar{S} = K$, where the option is always exercised if valuable, f simplifies to the BS price.

Ignoring the correlation between departure and the stock price, one would multiply the unconditional probability $\mathrm{E}[q(S)]$ by the BS price. The 'uncorrelated' option value is given by

$$f' = \mathrm{E}[q(S)]\cdot \mathrm{e}^{-r\tau}\mathrm{E}^*[\max(S - K, 0)]$$
$$= (\mathrm{e}^{\mu\tau}/\bar{S})[S_0 - c(S_0; \mathrm{e}^{(r-\mu)\tau}\bar{S}, r, \sigma, \tau)]\cdot c(S_0; K, r, \sigma, \tau). \tag{7}$$

[6] Any risk that the *actual* number of options granted is not equal to the *expected* number $q(S)$ is assumed to be diversifiable, and therefore not priced.

Note that because it involves an expectation over the stock price with no risk adjustment, the uncorrelated approach leads to an option price dependent on μ.

For the case $\bar{S} = K$, when $\mu = r$, the uncorrelated value f' is

$$\frac{S_0 - c(S_0; K, r, \sigma, \tau)}{e^{-r\tau}K} \cdot c(S_0; K, r, \sigma, \tau). \tag{8}$$

Since $c \geqslant S_0 - e^{-r\tau}K$, (8) is bounded above by the BS price c; thus, f' is lower than f. Intuitively, the valuation of f correctly recognizes that the departure probability is irrelevant if departure only occurs when the ESO expires worthless. When the ESO has any value, the executive stays; the correct option value is therefore the same as Black–Scholes. The uncorrelated approach does not recognize this. For example, with a ten-year ESO issued at-the-money with $r = 5\%$ and $\sigma = 30\%$, the uncorrelated value f' is only 78% of the correct (BS) value.

Generally, under the assumption of risk neutrality, uncorrelated valuation assuming exogenous departure underestimates the ESO value when the probability of departure is negatively related to the stock price.

4. Performance-based options

Performance-based ESOs grant the executive a variable number of options at the end of the vesting period, depending on the state of nature at that time. We consider vesting schemes where the number of granted options depends on the stock price at vesting. To distinguish between a usual option and a package with a variable number of options, we call the latter a 'grant'.

Consider a grant giving a number of portable European call options on a non-dividend-paying stock at vesting. The expected number of options given, $q(S)$, may depend on performance as measured by S. The grant value is therefore given by (2). When $q(S)$ is a power function, the grant value can be solved in closed-form.

Assume $q(S) = (S/\bar{S})^n$. In general, the parameters $\bar{S}$ and n depend on the grant provided by the firm. Ignoring the correlation between the number of options granted at vesting and their value, the uncorrelated valuation is

$$\begin{aligned} f' &= \mathrm{E}[q(S)] \cdot e^{-r\tau_1} \mathrm{E}^*[c(S; K, r, \sigma, \tau_2)] \\ &= \mathrm{E}[q(S)] \cdot c(S_0; K, r, \sigma, \tau) \\ &= (S_0/\bar{S})^n e^{[\mu + (n-1)\sigma^2/2]n\tau_1} \cdot c(S_0; K, r, \sigma, \tau). \end{aligned} \tag{9}$$

Recognizing this correlation between $q(S)$ and $c(S)$ leads to the following corrected valuation.

Proposition 1. The value of a performance-based compensation package paying $q(S) = (S/\bar{S})^n$ portable European call options at vesting is

$$f = (S_0/\bar{S})^n e^{[r + (n-1)\sigma^2/2]n\tau_1} \cdot c\left(S_0; K, r + n\sigma^2\frac{\tau_1}{\tau}, \sigma, \tau\right), \tag{10}$$

where S_0 is the current stock price and $\tau = \tau_1 + \tau_2$ is the time until expiration.

Proof. By risk-neutral valuation

$$f = e^{-r\tau}E\left[(S/\bar{S})^n\left(SN\left[\frac{\ln S/K + (r + \sigma^2/2)\tau_2}{\sigma\sqrt{\tau_2}}\right]\right.\right.$$

$$\left.\left. - e^{-r\tau_2}KN\left[\frac{\ln S/K + (r - \sigma^2/2)\tau_2}{\sigma\sqrt{\tau_2}}\right]\right)\right],$$

with $S = S_0 e^{(r - \sigma^2/2)\tau_1 + \sigma\sqrt{\tau_1}z}$ and z a standard normal random variable, and $N(\cdot)$ is the cumulative normal distribution function. This gives two integrals which can be evaluated by standard techniques. □

Even under risk neutrality, with $\mu = r$, recognizing this correlation is important in valuing ESOs. Comparisons between (10) and (9) are easily made, illustrating the effect of properly including the correlation between the number of options and their value. Consider for example an at-the-money ten-year grant, with a strike price of \$100, vesting at $\tau_1 = 3$ years, $r = 5\%$, and $\sigma = 30\%$. To compare these two methods, choose $\bar{S}$ so that the expected number of options granted is unity.

Under risk neutrality, when the firm expects to grant one option on a stock with no dividends, the uncorrelated approach sets the grant value to the BS price, \$52.57. Recognizing the correlation, however, leads to much higher values, \$65.03, when $q(S)$ varies with the square root of S, and \$79.65, when $q(S)$ is linear in S. Thus, when the number of options granted is a linear function of the stock price, the corrected grant value is 52% higher than that of the uncorrelated approach. Under other reasonable assumptions about the stock growth rate and dividend yield, the differences between the uncorrelated and corrected valuations remain substantial.

5. Numerical analysis

The analyses of Sections 3 and 4 illustrate the importance of the interactions between the stock price and both departure and the meeting of performance targets. The special cases described have the advantage of closed-form solutions, but the disadvantage of restrictive assumptions. The more general case should incorporate the possibility of early departure, as well as both vesting and non-portability restrictions.

To include these characteristics in a more general analysis, numerical methods are used. We use the binomial tree approach of Cox, Ross, and Rubinstein (1979), using n steps, with a span $\Delta t = \tau/n$. Define $i = 0, 1, \ldots, n$ as the index for the steps, and $j = 0, 1, \ldots, i$ as the index for the cross-section. Working backward through the tree, the option value can be determined at each node.

Early vesting and possibility of departure can be accounted for in the numerical analysis as follows. First, for $0 \leq i \leq n - 1$, compute the usual call option value as a function of its future values and the risk-neutral probability of an up-move π,

$$c^{i,j} = e^{-r\Delta t}[\pi \cdot f^{i+1,j+1} + (1 - \pi)\cdot f^{i+1,j}]. \tag{11}$$

Next, the executive leaves with probability $1 - e^{-\lambda\Delta t}$, forcing early exercise; the executive stays with probability $e^{-\lambda\Delta t}$, giving option value $c^{i,j}$. Thus, when the executive is vested,

$$f^{i,j} = (1 - e^{-\lambda\Delta t})\cdot \max(S^{i,j} - K, 0) + e^{-\lambda\Delta t}\cdot c^{i,j}, \tag{12}$$

while when the executive is not vested,

$$f^{i,j} = (1 - e^{-\lambda\Delta t})\cdot 0 + e^{-\lambda\Delta t}\cdot c^{i,j}. \tag{13}$$

Our contribution is to note that if the departure rate $\lambda(S^{i,j})$ depends on the stock price, the valuation will be affected.

The analogous continuous-time model has a price process (B_t, S_t), where the bond price B_t grows at rate r, and the stock price S_t follows a geometric Brownian motion with volatility σ. The ESO has an early exercise rate $\lambda(S_t)$ at time t.

Proposition 2. For λ a nonnegative continuous function, the discrete-time ESO price of (11) through (13) converges to the continuous-time ESO price as the number of steps $n \to \infty$.

Proof. See Appendix.

We calculate option values for various departure rates. As in Warner et al. (1988), we use a logistic function to relate the probability of departure to the stock price, and choose parameters to give (unconditional) departure rates varying from 2% to 10%.[7]

[7] The logistic function is $\lambda(R) = [1 + \exp(a + bR)]^{-1}$, where R is the excess return on the stock since time 0. The parameter b is chosen from Warner et al. as 2.31, and the parameter a is varied so as to achieve the desired unconditional departure rate.

Table 1

Valuation of executive stock options

The table compares, for various average departure rates and vesting periods, the price of European options obtained using (i) the 'corrected' (Corr.) method which accounts for interactions between the decision to leave and the price level, (ii) Jennergren and Näslund (JN) method (assuming a constant departure rate), and (iii) the FASB proposed method which adjusts by the probability of departure during the vesting period, and uses the expected term of the option. Parameters include: spot = 100, strike = 100, interest rate = 5%, dividend yield = 2%, time = 10 years, volatility = 30%.

Average departure rate	Vesting = Now			Vesting = 3 years			Vesting = 5 years		
	Corr.	JN	FASB	Corr.	JN	FASB	Corr.	JN	FASB
	Dollar price								
0%	37.65	37.65	37.65	37.65	37.65	37.65	37.65	37.65	37.65
2%	36.45	35.51	36.47	36.31	34.76	34.34	36.21	33.83	33.00
4%	34.78	33.58	35.28	34.19	32.13	31.29	34.19	30.40	28.89
6%	32.90	31.84	34.11	32.26	29.75	28.49	31.86	27.35	25.27
8%	30.93	30.27	32.97	29.97	27.57	25.93	29.39	24.61	22.10
10%	28.98	28.84	31.86	27.66	25.58	23.60	26.88	22.16	19.32
	Percentage deviation from BS								
2%	−3.2	−5.7	−3.1	−3.6	−7.7	−8.8	−3.8	−10.1	−12.3
4%	−7.6	−10.8	−6.3	−9.2	−14.7	−16.9	−9.2	−19.3	23.3
6%	−12.6	−15.4	−9.4	−14.3	−21.0	−24.3	−15.4	−27.4	−32.9
8%	−17.8	19.6	−12.4	−20.4	−26.8	−31.1	−21.9	−34.6	−41.3
10%	−23.0	−23.4	−15.4	−26.5	−32.1	−37.3	−28.6	−41.1	−48.7

We also compare our valuation method to the method proposed by the FASB. Under this method, the vesting restrictions are modelled by multiplying the option value by the probability of staying until the end of the vesting period. The nonportability restrictions are captured by using the *expected* option term, $\bar{\tau}$, in the standard BS model

$$f^{a} = e^{-\lambda\tau_1} c(S_0; K, r, \sigma, \bar{\tau}). \tag{14}$$

Table 1 compares ESO valuations obtained using the Jennergren and Näslund (JN) method and proposed FASB method (assuming a constant departure rate) with the corrected method (assuming a departure rate dependent on stock return). Parameters are: spot = strike = 100, interest rate = 5%, dividend yield = 2%, option life = 10 years, volatility = 30%. Vesting periods of zero, three, and five years are shown separately in three panels; ESOs typically vest at three or five years. When the departure rate λ is zero, the European binomial option price is $37.65.

Both the JN and FASB methods lead to a serious reduction in value. However, the corrected method yields reductions from BS about one-half to two-thirds that suggested by the JN and FASB methods. The table also shows that the biases of the JN and FASB methods increase as the vesting period increases.

It can be shown that allowing for American features does not appreciably change these results. As before, a negative correlation between departure and the value of the underlying stock increases the value of ESOs. A word of caution is in order for empirical research using ESOs with American features. With American options, dividend payments may induce rational early exercise, which should in no way decrease the value of the option. Given that the effect of early exercise for rational reasons is already captured in American option values, it is essential that empirical studies of early exercise of ESOs distinguish between rational exercise due to the American feature and exercise for other reasons.

6. Conclusions

Executive stock options differ from exchange-traded options because of vesting and portability restrictions. FASB proposal 127-C allows for adjustments to the Black–Scholes model, using the departure probability, expected option life, or (in the case of performance-based options) the probability of achieving a required objective.

This approach leads to sharp reductions in option values, but ignores important interactions between option values and departure decisions. Specifically, it ignores important incentive effects: executives are more likely to stay when stock prices are high. Because options involve nonlinear payoffs in underlying stock prices, ignoring the correlation between the number of exercised options and the stock price leads to incorrect valuation.

Based on results from earlier empirical studies, we find that ignoring the link between departure rates and the stock price leads to substantial undervaluation of ESOs. These interactions are particularly important for performance-based options, where more options are granted in states of the world where the firm prospers, which is when the stock price is high. When the number of granted options varies linearly with the stock price, usual methods can undervalue the true option value by over 50%.

Appendix: Proof of Proposition 2

We show that as the number of steps $n \to \infty$, the ESO price under the discrete-time model converges to the ESO price under the continuous-time model. Let S_t^n be the stock price, B_t^n the price of a riskless bond, and $\lambda(S_t^n)$ the exit (early exercise) rate at time t for the discrete model with n steps. The ESO vests at τ_1 and expires at τ. The usual n-step discrete time binomial price process (B^n, S^n) of Cox, Ross, and Rubinstein (CRR, 1979) is extended to all times $[0, \tau]$ by associating the value at time t with the node $[nt]/n$. Define the payoff of the ESO over a particular stock price path,

$$h(B^n, S^n) = \int_{\tau_1}^{\tau} (B_t^n)^{-1} \exp\left[-\int_0^t \lambda(S_u^n)\mathrm{d}u\right] \lambda(S_t^n)(S_t^n - K)^+ \mathrm{d}t + (B_\tau^n)^{-1} \exp\left[-\int_0^\tau \lambda(S_u^n)\mathrm{d}u\right](S_\tau^n - K)^+, \tag{15}$$

where $(S_t^n - K)^+ = \max(S_t^n - K, 0)$, and write ν_n for the process distribution. The value of the ESO for the n-step discrete model is then $\int h(B^n, S^n)\mathrm{d}\nu_n$. In all cases, dropping the n superscript will denote the corresponding continuous-time expression. We need to show

$$\int h(B^n, S^n)\mathrm{d}\nu_n \to \int h(B, S)\mathrm{d}\nu.$$

Note that

$$\int_{\tau_1}^{\tau} \exp\left[-\int_0^t \lambda(S_u^n)\mathrm{d}u\right]\lambda(S_t^n)\mathrm{d}t + \exp\left[-\int_0^\tau \lambda(S_u^n)\mathrm{d}u\right]$$

$$= \exp\left[-\int_0^{\tau_1} \lambda(S_u^n)\mathrm{d}u\right] \le 1.$$

Also note that, for any $A \geq 0$,

$$(S_t^n - K)^+ = \min[A, (S_t^n - K)^+] + (S_t^n - K - A)^+. \tag{16}$$

These will be useful in providing a bound on the difference of the discrete and continuous ESO values.

By substituting (16) into (15), and using the triangle inequality,

$$\begin{aligned}
&\left| \int h(B^n, S^n)\mathrm{d}\nu_n - \int h(B, S)\mathrm{d}\nu \right| \\
&\leqslant \left| \int \left(\int_{\tau_1}^{\tau} (B_t^n)^{-1} \exp\left[- \int_0^t \lambda(S_u^n)\mathrm{d}u \right] \lambda(S_t^n) \min[A, (S_t^n - K)^+]\mathrm{d}t \right.\right. \\
&\quad \left. + (B_\tau^n)^{-1} \exp\left[- \int_0^\tau \lambda(S_u^n)\mathrm{d}u \right] \min[A, (S_\tau^n - K)^+] \right) \mathrm{d}\nu_n \\
&\quad - \int \left(\int_{\tau_1}^{\tau} (B_t)^{-1} \exp\left[- \int_0^t \lambda(S_u)\mathrm{d}u \right] \lambda(S_t) \min[A, (S_t - K)^+]\mathrm{d}t \right. \\
&\quad \left.\left. + (B_\tau)^{-1} \exp\left[- \int_0^\tau \lambda(S_u)\mathrm{d}u \right] \min[A, (S_\tau - K)^+] \right) \mathrm{d}\nu \right| \\
&\quad + \left| \int \left(\int_{\tau_1}^{\tau} (B_t^n)^{-1} \exp\left[- \int_0^t \lambda(S_u^n)\mathrm{d}u \right] \lambda(S_t^n)(S_t^n - K - A)^+ \mathrm{d}t \right.\right. \\
&\quad \left.\left. + (B_\tau^n)^{-1} \exp\left[- \int_0^\tau \lambda(S_u^n)\mathrm{d}u \right] (S_\tau^n - K - A)^+ \right) \mathrm{d}\nu_n \right| \\
&\quad + \left| \int \left(\int_{\tau_1}^{\tau} (B_t)^{-1} \exp\left[- \int_0^t \lambda(S_u)\mathrm{d}u \right] \lambda(S_t)(S_t - K - A)^+ \mathrm{d}t \right.\right. \\
&\quad \left.\left. + (B_\tau)^{-1} \exp\left[- \int_0^\tau \lambda(S_u)\mathrm{d}u \right] (S_\tau - K - A)^+ \right) \mathrm{d}\nu \right|.
\end{aligned}$$

Of these four pairs of terms, the first is the expectation of a bounded, continuous function of (B^n, S^n), the second is the expectation of the same bounded, continuous function, now of (B, S). Since the CRR binomial process is weakly convergent (see He, 1990), the difference goes to zero as $n \to \infty$. The third pair is the value of the ESO (in discrete time) with strike price $K + A$. Since early exercise of non-dividend-paying calls is never optimal (see Merton, 1973), this is bounded above by the value of a European call (in discrete time) with parameters $(S_0, K + A, r, \sigma, \tau)$. As $n \to \infty$, this converges to the Black–Scholes call value $c(S_0, K + A, r, \sigma, \tau)$ (see CRR, 1979). Similarly, the fourth pair is the value of the ESO (in continuous time) with strike price $K + A$, bounded above by $c(S_0, K + A, r, \sigma, \tau)$.

Thus, for all $A \geq 0$, we have

$$\lim_{n \to \infty} \left| \int h(B^n, S^n) \mathrm{d}v_n - \int h(B, S) \mathrm{d}v \right|$$
$$\leq 0 + c(S_0, K + A, r, \sigma, \tau) + c(S_0, K + A, r, \sigma, \tau)$$
$$= 2c(S_0, K + A, r, \sigma, \tau).$$

Letting $A \to \infty$, c goes to 0. This proves the convergence result.

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